Exam

Electricity and Magnetism 1

Monday June 16, 2014
8:30-11:30

Read these instructions carefully before making the exam!

- **Write your name and student number on *every* sheet.**
- ***Write clearly*.**
- ***Language*; your answers have to be in English.**
- **Use a *separate* sheet for each problem.**
- **Use of a (graphing) calculator is allowed.**
- **This exam consists of 4 problems.**
- **All 4 problems are of equal weight. Weights of the various subproblems are indicated at the beginning of each problem.**
- **For all problems you have to write down your arguments and the intermediate steps in your calculations.**

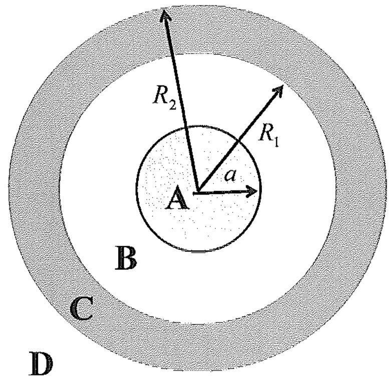
PROBLEM 1

Score: a+b+c+d+e+f+g=3+2+3+3+3+2+2=18

A solid plastic (non-conducting and not polarisable) sphere is positioned at the centre of a uncharged conducting hollow sphere with inner radius R_1 and outer radius R_2 (see figure). The radius of the plastic sphere is a and the sphere carries a volume charge density,

$$\rho(r) = \rho_0 \frac{e^{-\beta r}}{(\beta r)^2}$$

in which r is the radial coordinate and ρ_0 and β are positive constants.



a) Show that the total charge Q on the plastic sphere is $Q = \frac{4\pi\rho_0}{\beta^3}\left(1 - e^{-\beta a}\right)$.

We subdivide all space in four regions A, B, C and D (see figure):
A: $r \leq a$; B: $a < r < R_1$; C: $R_1 \leq r \leq R_2$; and D: $r > R_2$.

b) Write down Gauss's law for the electric field in integral form.

For c) to g) express yours answers in terms of Q in favour of ρ_0 by using the relation from a).

c) Find the electric field $\vec{E}$ in the regions A, B, C and D.
d) Find the potential at the *surface* of the plastic sphere. Set the zero of the potential at infinity.

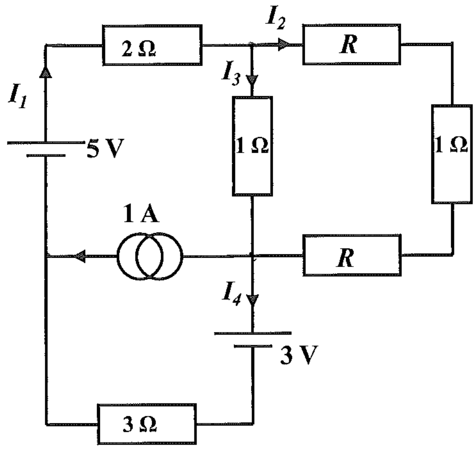
Now consider the situation in which region B is filled with a linear dielectric with dielectric constant ε_r.

e) Find the electric field $\vec{E}$ in the regions A, B, C and D.
f) Find the polarization $\vec{P}$ in region B.
g) Find both the bound volume charge density and the bound surface charge density in the dielectric.

PROBLEM 2

Score: a+b+c+d+e=4+3+4+4+3=18

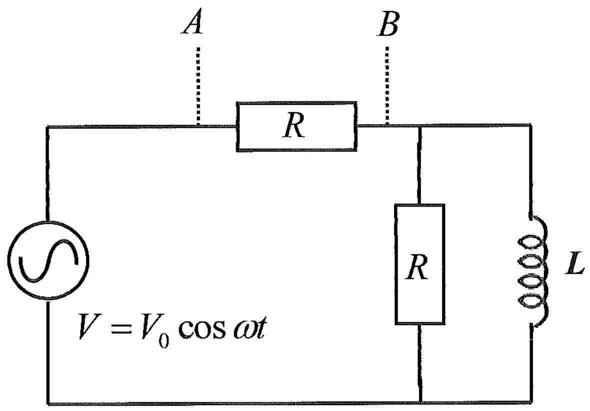
Consider the electric circuit in the figure below.



a) Find all the node equations (Kirchhoff 1). Show that one of these equations can be derived from the other equations
b) Find all the loop equations (Kirchhoff 2).
c) Suppose $I_1 = 4I_2$. Find the value of the resistance R.

Consider the electric circuit in the figure below. The stationary voltage source is described (in the real representation) by $V = V_0 \cos(\omega t)$.

d) Find the potential difference $V_{AB} = V_B - V_A$ over the resistor (see figure) in the complex representation.
e) Find the real potential difference $V_{AB} = V_B - V_A$ over the resistor.



PROBLEM 3

Score: a+b+c+d+e+f+g+h=1+3+2+2+3+2+3+2=18

General: *Use cylinder coordinates. All edge effects may be neglected.*

A long coaxial cable consists of a solid inner cylindrical conductor of radius a, surrounded by a concentric cylindrical tube of inner radius b and outer radius c (see figure). The solid inner conductor carries a volume current density $\vec{J}_1 = J_1 \frac{s\hat{z}}{a}$ with s the radial coordinate. The outer tube carries a current $\vec{I}_2 = -I_2\hat{z}$. This current is distributed uniformly across the cross-section of the tube.

a) Give the units of J_1 and I_2.

b) Proof that the total current through the solid inner conductor is:

$$\vec{I}_1 = \frac{2}{3}\pi a^2 J_1 \hat{z}$$

c) Derive an expression for the volume current distribution $\vec{J}_2$ through the outer cylindrical tube.

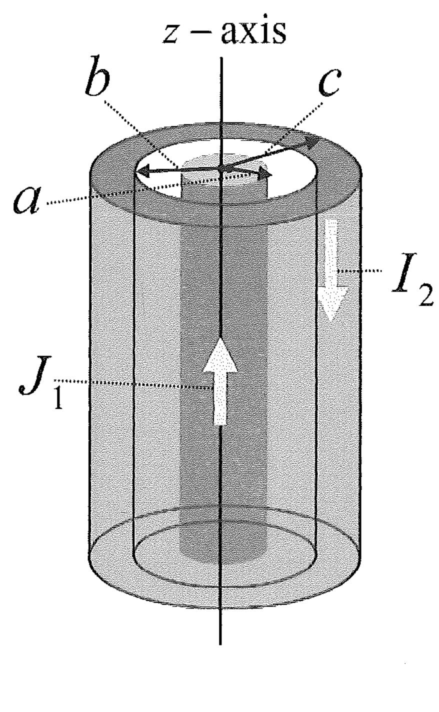
d) Write down Ampère's law for the magnetic field in integral form.

e) Find the magnetic field $\vec{B}$ in the region $0 \leq s \leq a$.

f) Find the magnetic field in the region $a < s < b$.

g) Find the magnetic field in the region $b \leq s \leq c$.

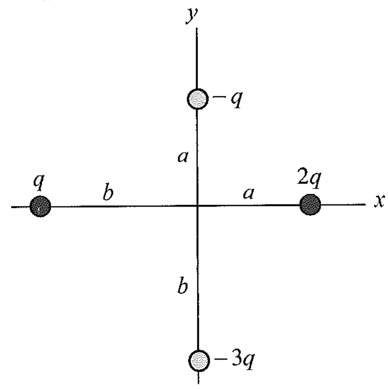
h) Find the ratio $\frac{I_2}{J_1}$ such that $\vec{B} = 0$ in the region $s > c$.



PROBLEM 4

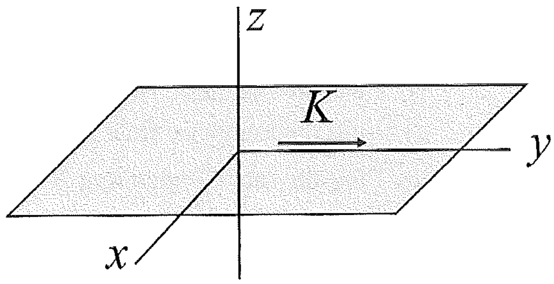
Score: a+b+c+d =4+5+5+4=18

Consider a charge distribution (see figure) in the xy-plane consisting of two positive and two negative charges. The positive charges have magnitude q and $2q$ and are located on the x-axis at $x = -b$ and $x = a$, respectively. The negative charges have magnitude $-q$ and $-3q$ and are located on the y-axis at $y = a$ and $y = -b$, respectively.



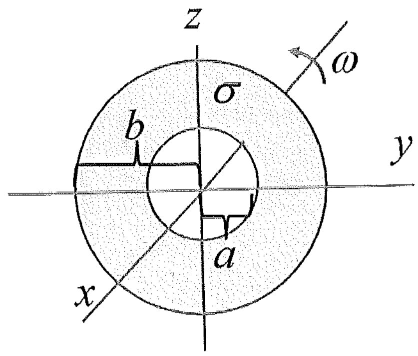
a) Find the work that is needed to make this charge distribution. Express your answer in terms of the charge q and the distances a and b.
b) Find the electric monopole and dipole moment of this charge distribution.

A sheet in the xy-plane carries a surface current $\vec{K} = K\hat{y}$ (see figure).



c) Find the vector potential $\vec{A}$ above and below the sheet. You may use your knowledge of the magnetic field of the sheet.

Consider a disk with surface charge density σ, inner radius a and outer radius b (see figure). The disk rotates counter clockwise around the x-axis with angular velocity ω.



d) Find the magnetic dipole moment $\vec{m}$ of this disk.